3.1 Interpreting Distance versus Time Graphs

Overview: Participants use motion detectors to investigate distance over time graphs. This lays the groundwork for graph reading and for work with rates of change.

Objective: Algebra I TEKS
(b.2.C) The student interprets situations in terms of given graphs or creates situations that fit given graphs.

Terms: rate of change, increasing, decreasing, constant

Materials: motion detector connected to an overhead calculator, motion detectors, data collection devices, graphing calculators, 2 or 3 transparencies cut-to-fit on the overhead calculator screen

Procedures: This activity is done in an open area with room for participants to move about in groups of 2 – 4.

Have participants complete the Student Activity: Walk This Way. As you do the whole group introduction, sketch two of the walks of the participants. Do this by placing a transparency, cut to fit, on the overhead calculator screen and then sketch the walk with a transparency marker. We will use these sketches of walks at the end of Activity 2.

Math Note: Scientific convention is to write $y$ versus $x$ when referring to a situation of $(x, y)$. In other words, the dependent variable is always listed first, then the independent variable as in: dependent variable versus independent variable. Thus, this activity refers to Distance versus Time graphs or Distance over Time graphs, where time is the independent variable and distance is the dependent variable.

Activity 1: Walking Graphs
Have participants answer the questions in their groups. Circulate and answer questions. Ask a member of each group to present an answer for one of the Exercises.

Answers will vary. Sample answers:
1. The $y$-intercept of the first segment is the distance away from the motion detector at time $= 0$. This tells you where to start walking.
2. The $x$-axis represents time and each tick mark represents a second. The number of tick marks on the $x$-axis represents the time to walk for each segment.
3. If the segment decreases as time increases on the $x$-axis, then the distance from the motion detector is decreasing. This means you should walk toward the motion detector.
4. If the segment increases as time increases on the $x$-axis, then the distance from the motion detector is increasing. This means you should walk away from the motion detector.

5. If the segment decreased or increased slowly (it was shallow), then that means that the distance from the motion detector was changing slowly as time increased. Therefore, you should walk slowly.

6. If the segment decreased or increased rapidly (it was steep), then that means that the distance from the motion detector was changing rapidly as time increased. Therefore, you should walk quickly.

7. If the segment was horizontal, then that means that the distance from the motion detector was not changing as time increased. The distance was remaining constant. Therefore you should stand still.

Put up the transparency of the student activity: Practice Walking Linear Graphs. Draw triangles on some of the line segments to show that each line segment has a constant rate of change, that for every increment in the $x$-direction, the segment increases or decreases by the same amount in the $y$-direction. Do not spend too much time here. Simply demonstrate that for a given line segment, triangles drawn as shown with equal bases are congruent. This means that the heights are equal. In the next activity, we will contrast these constant rates of change with non-constant rates of change of non-linear graphs.

**Activity 2: Walking More Graphs**

In this activity, participants walk non-linear graphs. Have participants collect data for about 4 seconds. (This may necessitate collecting the data not in real-time. If participants prefer to collect data in real-time (seeing the data as they collect it), simply have participants ignore the tick marks on the $x$-axis and collect data for 15 seconds. Discuss the advantages and disadvantages of both methods. In real time, participants can quickly adjust their motion in the middle of data collection. However, that may mean that they do not analyze and plan carefully first. Out of real time, participants must analyze and plan carefully. Then after obtaining the resulting graph, they adjust the plan and walk again.

The big idea in the activity is to get a feel for changing rates of change, therefore the exact starting and stopping points are less important then the general shape of the graph.
Have participants do the activity and answer the questions in their groups. Circulate and help. Ask a member of each group to present an answer for one of the Exercises.

Answers will vary. Sample answers:

1. Start 6 feet from the motion detector. Stand still for less than a second. Start walking toward the motion detector slowly at first and then speeding up over 4 seconds.

2. Start about 2.5 feet from the motion detector. Start walking away quickly, slowing down over 4 seconds to a dead stop at the end.

3. Start about 2 feet from the motion detector. Start walking away slowly, speeding up over 4 seconds.

4. Start about 6 feet from the motion detector. Start walking toward the motion detector quickly, slowing down over 4 seconds to a dead stop at the end.

5. Start about 6 feet from the motion detector. Start walking toward the motion detector quickly, slowing down over 2 seconds, stop briefly, then walk away from the motion detector slowly, speeding up till the end of the four seconds.

6. Start about 2 feet from the motion detector. Start walking away from the motion detector quickly, slowing down over 2 seconds, stop briefly, then walk toward the motion detector slowly, speeding up till the end of the four seconds.

7. You know to speed up (accelerate) when the curve gets steeper, when the change in $y$ over an increment in $x$ is greater than it was before.

8. You know to slow down (decelerate) when the curve gets less steep, when the change in $y$ over an increment in $x$ is less than it was before.

9. e, f, g. These graphs show walking at a constant rate because the change in the $y$-direction is constant for a constant change in the $x$-direction. The rate of change is constant. For the horizontal line, the rate of change is a constant zero. Draw triangles to illustrate.
10. a,c. These graphs show a walker speeding up because the change in $y$ is increasing over a constant change in $x$. The rate of change is increasing. Draw triangles to illustrate.

![Graphs showing a walker speeding up.]

You can also draw tangent lines to illustrate that the slopes of the tangent lines (the instantaneous rate of change at the point of tangency) are increasing.

11. b,d. These graphs show a walker slowing down because the change in $y$ is decreasing over a constant change in $x$. The rate of change is decreasing. Draw triangles to illustrate.

![Graphs showing a walker slowing down.]

You can also draw tangent lines to illustrate that the slopes of the tangent lines (the instantaneous rate of change at the point of tangency) are decreasing.

12. e. As shown above, the rate of change for a horizontal line is a constant zero. The walker is standing still because for every change in $x$, the change in $y$ is a constant zero. The distance from the motion detector is not changing.
Can you produce a vertical line by walking in front of a motion detector? [No, a vertical line suggests that at one instant in time, the walker is at an infinite number of locations, every point on that line. A walker can only be one distance from the motion detector at a time.]

Wrap up these activities as a whole group using the sketches of participants’ walks on the cut-to-fit transparencies from the beginning. Put a sketch on a cut-to-fit transparency on the overhead calculator. (Consider using a walk that has varied sections, linear and non-linear.) Ask participants to discuss in their groups, how a walker should walk to reproduce the walk. Randomly choose a group’s description and have a walker walk the description to confirm. Repeat if desired.

Sample Answers to Reflect and Apply:

1. I started jogging away from my house (when I was 0.5 kilometers from my house) and I gradually sped up to a sprint. At 15 minutes (tick marks are at 5 minute increments), I fell on the ground, 4 kilometers from my house. I sat there for about 20 minutes, catching my breath. I then started gradually back home, speeding up as I went until I was again sprinting in the door.

2. I walked from my locker to my class over two minutes. I stood there chatting for a minute until I realized I had left my book in my friend’s locker. I walked quickly to my friend’s locker, which was thankfully open, grabbed the book and started quickly back to class. But I ran into the principal who walked me back to class, slowing down as we went, because the principal was talking to me about an upcoming event.

Summary: The big idea here is that walking at a constant rate produces a linear distance over time graph. The rate of change of a line is constant. Speeding up or slowing down, non-constant rates of change, produce non-linear distance over time graphs. The rates of change are changing.
Activity 1: Walking Graphs

Answer the following questions based on your experiences in the Student Activity: Walk This Way.

1. How did you know where to start walking for each graph?

2. How did you know how long to walk for each segment?

3. How did you know when to walk toward the motion detector? Use the words “time” and “distance” in your answer.

4. How did you know when to walk away from the motion detector? Use the words “time” and “distance” in your answer.

5. How did you know when to walk slowly?

6. How did you know when to walk quickly?

7. How did you know when to stand still?
Activity 2: Walking More Graphs

Practice walking the following graphs using a motion detector and a graphing calculator. Describe the walk that you used to produce each graph.

1. 
2. 
3. 
4. 
5. 
6.
7. How did you know when to speed up?

8. How did you know when to slow down?

Which graph(s) below show:
9. a constant rate? Why?
10. a walker speeding up? Why?
11. a walker slowing down? Why?
12. a walker standing still? Why?

![Graphs a to g](image-url)
Reflect and Apply

Make up a story for the following graphs of distance over time.

1. 

2. 